

# A Combined Approach for Master-slave Robotic Surgery System Control Based on Jacobian and PD Algorithm\*

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**Abstract** - In this paper, a semi-physical simulation platform of robot teleoperation system is designed, and a combined control approach using Jacobian matrix and Proportional-Derivative (PD) algorithm is proposed. A digital low-pass filter is designed to eliminate noise signal caused by hand-vibration of surgeons. The advantages of the proposed approach include easy implementation, real-time response and accurate master-slave tracking. The results of semi-physical simulation validated that the proposed approach realized accurate master-slave tracking for puncture surgery and the noise caused by hand-vibration was eliminated effectively.

**Keywords**—robotic puncture surgery; teleoperation; Jacobian matrix; PD control algorithm; vibration elimination

## I. INTRODUCTION

Puncture surgery is a surgical operation that surgeon insert special surgical instruments into the patient under the guidance of medical images and perform the surgery through these instruments. As it could significantly reduce patient's pain, shorten recovery time and hospital stays, and relative research has become a competitive focus in many advanced nations [1]. At present, several medical teleoperation systems have been developed, such as AESOP system [2], ZEUS system [3] and DaVinci system [4]. This system not only provides surgeon with advanced visual feedback, but also endow the surgeons with flexible operation. They are milestone of medical surgical robots research.

The robot teleoperation system for puncture surgery is usually configured with a master-slave control way. In recent years, with the fast development of the relative technology, the isomeric master-slave robot gradually replaced the isomorphic master-slave robot in the field of puncture surgery. Robot kinematics and inverse kinematics, error elimination and vibration elimination must be finished in a short time to meet the real time control of the isomeric master-slave robot system.

In this paper, a combined method using Jacobian matrix and Proportional-Derivative (PD) control algorithm is proposed to improve the accuracy and efficiency of slave robot in tracking master's motion. The surgeon can accurately conduct puncturing operation by applying this combined method. The advantage of the proposed method is that it can achieve accurate tracking and quick response, and can be easily implemented in surgery robotic systems. Semi-physical simulation results validated the superiority of the proposed method. The remainder

of this paper is organized as follows. Section II introduces the hardware of the robot teleoperation system; Section III describes the teleoperation system of the puncture surgery robot and the control strategy; Section IV describes the semi-physical simulation results; Section V concludes the paper.

## II. STRUCTURE OF THE TELEOPERATION SYSTEM

In this system, the master robot is Phantom Omni haptic device with 6 degree of freedom (DOF) manufactured by Sensable Technologies [5] (see Fig.1). Although all joints are rotational, the position of the end-effector is determined by the first three joints, and the orientation is determined by the last three joints. Note that the origin of the end-effector is point  $P$ . In this paper, all coordinate systems are under Cartesian space.

The slave robot is a 6 DOF manipulator, three for position and the other three for orientation (see Fig.2). The slave robot is used as the actuator of this system to realize operation.

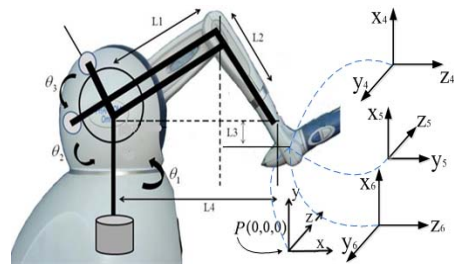


Fig.1 Phantom Omni robot and its initial position

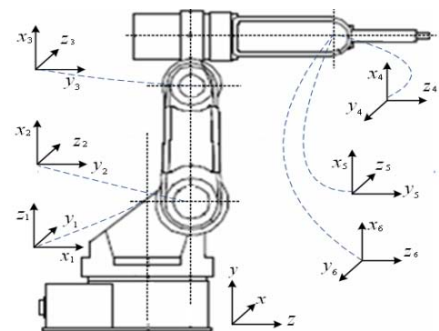


Fig.2 Slave robot coordinate configurations

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### III. MASTER-SLAVE CONTROL SYSTEM

The master-slave control system is a core portion of medical teleoperation system since it provides functions including coordinating master and slave robot, monitoring patient, and providing operation information for surgeons [6]. When a surgeon operates the master robot, information including the position, posture and velocity are transmitted to the slave robot via this control system. The parameters of PD controller are configured based on the feedback information to make sure that the slave robot can emulate the behavior of the master precisely and quickly.

Master-slave control system of puncture surgical robot is schematically illustrated in Fig.3.  $\mathbf{X}_m$  and  $\mathbf{X}_s$  denote the end-effector position vectors of master and slave robots, respectively.  $\dot{\mathbf{X}}_m$  and  $\dot{\mathbf{X}}_s$  denote the end-effector velocity vectors in of master and slave robots, respectively.  $\boldsymbol{\theta}_s$  and  $\dot{\boldsymbol{\theta}}_s$  denote the joint velocity vectors of master and slave robots.  $\mathbf{J}_s^{-1}$  denotes the inverse Jacobian matrix of the slave robot.  $k$  denotes the proportional coefficient of the master-slave mapping.

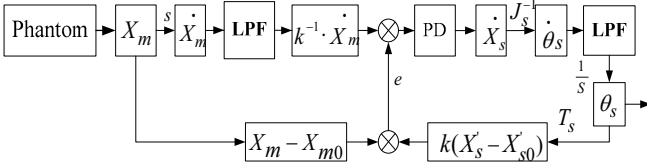


Fig.3. Block diagram of Master-Slave robot control system

#### A. Master-slave Control Method

In general, the control of master-slave system is determined by joint structure of master and slave robot. In the case that master and slave robot have the same structures, kinematic equations of the two robots also are the same. The slave robot can emulate the behavior of the master robot precisely and quickly using a simple joint-joint control method [7]. In another case that master and slave robots have heterogeneous mechanical structures, joints of master and slave robot are not in a one-to-one relationship. The system cannot be controlled using the joint-joint control method. Based on it, forward and inverse kinematics has to be involved to build a mapping between master and slave robots through Jacobian matrix in this paper.

Solution of inverse kinematics problem is the key of Master-slave tracking control. Many methods are available to get the solution, such as Lee and Ziegler's geometry algorithm [8] and Paul's counter transformation method [9] et al. However, computation of these methods is complex due to transcendental function in joints variable expression. And the optimal solution needs to be chosen from multiple solutions, which reduces the real-time performance of the system. In addition, solving inverse kinematics becomes more difficult if considering error feedback loop. In this paper, by applying differential transformation method, which uses displacement of tiny period of time instead of instantaneous velocity of joint, and the unique solution is derived when solving inverse kinematics. In this way, the calculation complexity is reduced and real-time performance of the system is improved.

Based on the above analysis, we can get a mapping between the joint velocity and end-effector velocity using Jacobian matrix, which is expressed as:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \\ w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & \cdots & J_{16} \\ J_{21} & J_{22} & \cdots & J_{26} \\ \vdots & \vdots & \ddots & \vdots \\ J_{61} & J_{62} & \cdots & J_{66} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix}, \quad (1)$$

and (1) can be simplified as:

$$\dot{\mathbf{X}} = \mathbf{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}, \quad (2)$$

where  $\dot{\boldsymbol{\theta}} \in \mathbf{R}^{6 \times 1}$  denotes the joint velocity vector,  $\mathbf{J}(\boldsymbol{\theta}) \in \mathbf{R}^{6 \times 6}$  denotes the Jacobian matrix of the robot, and  $\dot{\mathbf{X}} \in \mathbf{R}^{6 \times 1}$  denotes the end-effector velocity vector.  $J_{ij}(\boldsymbol{\theta}) = \frac{\partial X_i}{\partial \theta_j}$ ,  $i=1, 2, \dots, 6, j=1, 2, \dots, 6$ , where  $X_i$  denotes the  $i$ -th element of the end-effector position vector,  $\theta_j$  denotes the  $j$ -th element of the joint angle vector.  $\dot{\mathbf{X}} = [\mathbf{v} \ \mathbf{w}]^T$ , where  $\mathbf{v} = [v_x \ v_y \ v_z]^T$  is the components of the linear velocity, and  $\mathbf{w} = [w_x \ w_y \ w_z]^T$  is the components of the angular velocity.

In the Jacobian matrix, the first three rows is transmission ratio of linear velocity of end-effector, the last three rows is transmission ratio of angular velocity of end-effector. On the other hand, each column of the Jacobian matrix describes the influence of the relative joint on the components of the end-effector velocity.

Using displacement of tiny period of time to instead of instantaneous velocity of joint, the relation between the end-effector velocity vector  $\Delta \mathbf{X}$  and the joint velocity vector  $\Delta \boldsymbol{\theta}$  can be expressed as

$$\Delta \boldsymbol{\theta} = \mathbf{J}^{-1}(\boldsymbol{\theta}) \cdot \Delta \mathbf{X}, \quad (3)$$

where,  $\mathbf{J}^{-1}(\boldsymbol{\theta})$  denotes the inverse Jacobian matrix.

The master end-effector velocity vector can be calculated by equation (2), and the slave end-effector velocity vector can be obtained through master-slave mapping. Then the joint velocity vector of slave robot can be calculated using equation (3). However, the inverse Jacobian matrix is a transformation matrix relative to the local space, and the master-slave tracking error will constantly accumulate with the movement of both robots, which will reduce the master-slave tracking accuracy greatly. To decrease the error, a PD controller is introduced into the control system.

In the PD controller, the proportional link corresponds to the master-slave position tracking error, and the derivative link corresponds to the master-slave velocity tracking error. The master-slave position error  $e$  is difference between the position  $\mathbf{X}_m$ , i.e. the motion vector of the master robot end-effector, and the position  $\mathbf{X}_s$ , i.e. the motion vector of the slave robot end-effector. In addition, this paper presents an increment control method to assist the master-slave mapping method to complete conveniently the operation. We need to set the end-effector initial position  $\mathbf{X}_{m0}$  and  $\mathbf{X}_{s0}$  of master and slave robot,

respectively. The master-slave position error can be expressed as

$$\mathbf{e} = (\mathbf{X}_m - \mathbf{X}_{m0}) - \mathbf{k}(\mathbf{X}_s - \mathbf{X}_{s0}), \quad (4)$$

where  $\mathbf{k}$  denotes the proportional coefficient of master-slave mapping.

The master-slave velocity error  $\dot{\mathbf{e}}$  is difference between the end-effector velocity  $\dot{\mathbf{X}}_m$  of the master robot and the end-effector velocity  $\dot{\mathbf{X}}_s$  of the slave robot, and it is expressed as

$$\dot{\mathbf{e}} = \dot{\mathbf{X}}_m - \mathbf{k}\dot{\mathbf{X}}_s, \quad (5)$$

In order to achieve the accurate master-slave tracking, we can get the basic tracking expression as below:

$$\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} = \mathbf{0}, \quad (6)$$

where  $\mathbf{K}_p$  and  $\mathbf{K}_d$  denote the proportional coefficient and derivative coefficient, respectively. Then the integrated tracking expression can be deduced from equation (4), (5) and (6):

$$\mathbf{K}_p ((\mathbf{X}_m - \mathbf{X}_{m0}) - \mathbf{k}(\mathbf{X}_s - \mathbf{X}_{s0})) + \mathbf{K}_d (\dot{\mathbf{X}}_m - \mathbf{k}\dot{\mathbf{X}}_s) = \mathbf{0} \quad (7)$$

From the above equation, we can get the end-effector velocity  $\dot{\mathbf{X}}_s$  of slave robot:

$$\dot{\mathbf{X}}_s = \mathbf{k}^{-1} \dot{\mathbf{X}}_m + (\mathbf{K}_d \mathbf{k})^{-1} \mathbf{K}_p ((\mathbf{X}_m - \mathbf{X}_{m0}) - \mathbf{k}(\mathbf{X}_s - \mathbf{X}_{s0})) \quad (8)$$

### B. Master-slave mapping method

During puncture surgery, surgeon needs to achieve millimeter-level operation using puncture surgery robot. Thus, appropriate ratio coefficient  $\mathbf{k}$  of master-slave mapping need to be configured, and increment control method assisting the master-slave mapping in accomplishing precise operation needs to be constructed.

The purpose of master-slave mapping is to obtain the ratio of master-side velocity to slave-side velocity of robotic end-effector. The ratio coefficient  $\mathbf{k}$  of the master and slave robot is an important parameter of system, which enables the slave robot to accomplish precise operation through the motion of the master robot. And the relationship between master and slave side can be expressed as:

$$\dot{\mathbf{X}}_s = \mathbf{k}^{-1} \dot{\mathbf{X}}_m, \quad (9)$$

where  $\mathbf{k} = \text{diag}(k, k, k)$ .

With a larger coefficient  $\mathbf{k}$ , operation of slave robot is more precise since the working space of slave robot is smaller than that of master robot. But with a too large coefficient  $\mathbf{k}$ , operation of the slave robot may fail due to too small working space of the slave robot.

Since the workspace of the slave robot is much larger than that of the master, the master robot may have arrived at its extreme position while the slave robot does not arrive at the destination location during the operation. This situation would be more serious if  $\mathbf{k}$  is set to be larger than 1. To avoid this problem, an increment control method is introduced to assist the master-slave mapping in accomplishing precise operation. Applying this proposed method, when the end-effector of master robot arrive at extreme position, operator can cut off the master-slave mapping and adjust the master robot to appropriate

position, and then operation is continued to implement. In this way, the end-effector of slave robot can arrive at any position within its work-space.

### C. The vibration elimination method

Hand-vibration of surgeon is unavoidable during puncture surgery, particularly, when the operation takes a long time. Unserviceable vibration is reflected in movement of the slave robot through masters-slave mapping, and would ultimately reduce operation accuracy or even operation failure [10]. The moving-average filter algorithm is introduced to eliminate the side effect of Hand-vibration. After filtering for the master-side, the slave-side signal is run back to the filter again. Then the vibration will be eliminated effectively.

Moving-average algorithm has a good performance for eliminating periodic noise. This algorithm improves greatly the calculating speed for sampling data and meets the real-time demands of the system, and makes the digital filtering easier to be implemented. The algorithm is described as follows:

$$N'_i = \frac{n_i + n_{i-1} + \dots + n_{i-n+1}}{n}, \quad (10)$$

where,  $i$  denotes a sampling period,  $n$  denotes the order of filter ( $i \geq n$ ), and  $n_i$  denotes the sample value,  $N'_i$  denotes filtering result.

## IV. EXPERIMENTS

In order to validate the performance of the proposed method, we used master-slave control scheme to execute the semi-physical simulation of master-slave position tracking. The simulation experiments were run on MATLAB Simulink with the Phansim toolbox in real-time. Phansim toolbox built up interface based on the OpenHaptics Toolkit. User can input torque or force into the box and read joint angle, gimbal angle, and gimbal position [11].

### A. Simulation of error elimination

Fig.4 shows the comparison of master-slave tracking error of the control systems with and without error feedback link. It validates that the control system with error feedback and PD controller the tracking error can be eliminated effectively and the system get much better performance regarding error elimination.

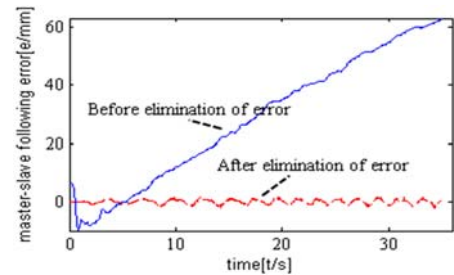


Fig.4 Master-slave tracking error before and after the application of proposed PD controller

### B. Simulation of vibration elimination

We set  $n=15$ , i.e. the filter is 15 order, to assess the performance of the vibration elimination. Fig.5 shows the vibration elimination results of slave end-effector. The results

from the semi-physical simulation indicate that vibrations in all axes are eliminated effectively, which validates that the filter can improve the system dynamic performance significantly.

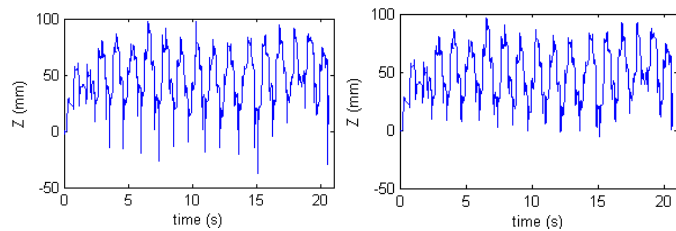


Fig.5 The typical slave end-effector trajectory before (left) and after (right) the application of trembling elimination

### C. Simulation of the master-slave tracking control method

In this semi-simulation, we implement the position tracking between master and slave robot by moving the master robot while the slave follows unconstrained. The proportional coefficient of the master-slave mapping is set to  $k=1$ , that is the motions of the slave are the same as the master. By tuning the control parameters, the tracking effect is good while the coefficient  $K_p = 0.3$  and  $K_d = 0.001$  of PD controller. the typical master-slave end-effector tracking trajectories and the tracking error are shown in Fig.6.

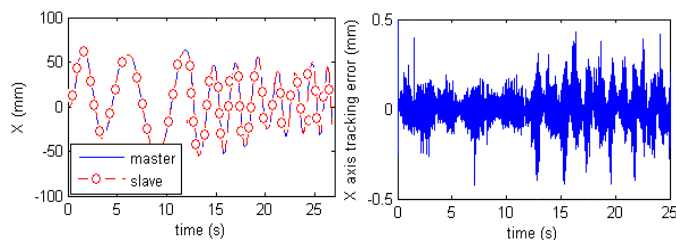


Fig.6 The typical master-slave position tracking result (left) and the tracking error (right)

Fig.6 shows that the slave robot can follow the movement of the master robot accurately and quickly in x-axis no matter the master robot moves on high speed or low, and the tracking error is less than 0.4 mm. And other axes also have similar performance with the x-axis. The results meet the requirements of the puncture surgery.

### D. Simulation of the master-slave robot nearly-conical pendulum motion tracking

The semi-physical simulation of the master-slave control system is continued to execute through changing proportional coefficient  $k$  of master-slave mapping. Fig.7 and Fig.8 show the semi-physical simulation results of nearly conical pendulum tracking with the coefficient  $K_p = 0.3$  and  $K_d = 0.001$ , when  $k$  is set to be 0.5 and 5, respectively. The tracking error calculation shows that the tracking errors are both less than 0.4 mm when  $k$  is set to be 0.5 and 5, respectively. These results can also meet the requirements of the puncture surgery, and the robot teleoperation system is relatively robust on different PD coefficients and different proportional coefficients.

## V. CONCLUSION

This paper proposed a combined control approach using Jacobian matrix and PD method for teleoperated master-slave

robotic system control. The proposed controller is easy to setup, independent of extra sensors, and is capable of both real-time responding and accurate tracking. The controller was validated in the semi-physical mode. In future, the controller will be applied in a real-life experimental platform using our slave robot prototype, and the dynamics and virtual fixtures will be studied to improve the human-robot collaborative performance.

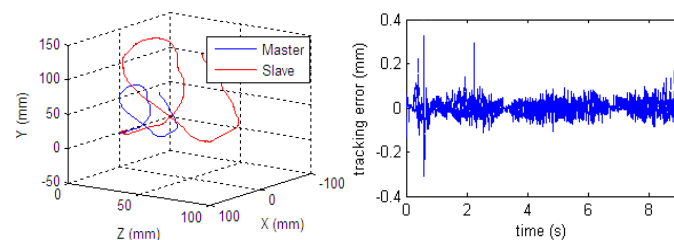


Fig.7. The typical nearly-conical pendulum tracking result (left) and tracking error (right) when  $k=0.5$

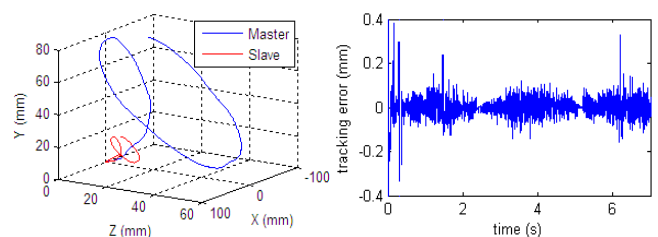


Fig.8. The typical nearly-conical pendulum tracking result (left) and tracking error (right) when  $k=5$

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